DOCUMENT RESUME

ED 296 870 SE 049 227

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TITLE Mathematics Curriculum Outline and Sample Activities for Gifted Students Grades 2 through 8.
INSTITUTION Mathematics Education Trust, Reston, VA.
PUB DATE 83
NOTE 40p.; Drawings may not reproduce well.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Class Activities; Computer Assisted Instruction; Computer Oriented Programs; Elementary Education; *Elementary School Mathematics; Functions (Mathematics); Geometry; *Gifted; Logic; *Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; Probability; Statistics

ABSTRACT

This document includes a statement of 11 basic considerations for a mathematics curriculum for the gifted in grades two through eight and a series of activities appropriate for such a curriculum. The activities are grouped under five strands: (1) logic; (2) combinations (combinatorics), probability, introduction to statistics; (3) geometry, topology; (4) function; and (5) computer. Each strand is introduced with a list of materials or embodiments to be used in instruction, student objectives, and the mathematical topics to be studied. The topics include but are not limited to the Fibonacci Sequence, analogies, Pascal's Triangle, composition of functions, and Monte Carlo methods. (PK)

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MATHEMATICS EDUCATION TRUST

Mathematics Curriculum Outline and Sample Activities for Gifted Students Grades 2 through 8

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These materials have been produced with financial support from the Mathematics Education Trust. Any opinions, findings, and conclusions or recommendations expressed are those of the writers and do not necessarily reflect the views of the Mathematics Education Trust.

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Reston, Virginia 22091

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MATHEMATICS CURRICULUM FOR THE GIFTED

Basic Considerations: Grades 2-8

A. The material presented herein has been developed on the basis of a number of considerations arising from research on the gifted. Most basic for the design is the Enrichment Triad Model (Renzulli, 1976) with its three types of enrichment activities. While the enrichment activities are not exhibited within the strands presented in this document, samples of such activities are available to teachers who must implement the program.

B. The strands developed here are to be adjoined to an existing mathematics program used in the Philadelphia School District by all schools in grades two through eight.

C. The complete program provides acceleration and enrichment by virtue of the augmented course of study and the specific design of implementation used in the Philadelphia School District. The student in this program would have to cover the basic material in less time, while devoting one day per week to the enrichment strands.

D. The program advocates the use of the content of mathematics as the raw material for creative acts and experiences. This advocation is based on the belief that doing mathematics provides the proper environment for learning mathematics.

E. The function of the program is to develop the intellectual abilities of the students, not necessarily to develop new content.

F. While the strands presented appear as a topical outline of the "natural" development of the subject matter, they do not represent a developmental map of the intellectual ordering of the material.

G. The material is also developed on the belief that mathematics is learned through cycles consisting of experience; abstraction of the situation; representation of the experience; internalization of the experience in linguistic, imagery, or symbolic form; and construction of organizing schema for analagous situations.

H. While the material is presented in a hierarchical organization, the instructional sequence is not mandated. Decisions as to instructional sequence must be left to the student evaluator. The numbers appearing after each line in the topical outline indicate a recommended mastery level in the Philadelphia Levels Program. This is to be used as a guide for instructional sequence.

I. Certain areas of mathematics were deemed mandatory in their inclusion in a program for the gifted. In addition to the strands presented here, it was felt that Applications, Geometry, Problem Solving and Calculators should be a part of the program. Since these elements exist as either separate strands in the basic Philadelphia program or are imbedded in one or more strands, separate development was not deemed necessary.

J. The enrichment strands, along with the basic program constitute a course of study which is defined by the following strands:

1. Numeration
2. Operations/Whole Numbers
3. Operations/ Rationals
4. Organization of data
5. Measurement
6. Geometry
7. Problem Solving
8. Functions
9. Computer Literacy/Programming
10. Logic
11. Combinatorics/Probability
12. Topology/Other Geometries

K. Finally, the material developed here is specifically designed for a population identified as gifted who will begin the program in second grade and follow it through eighth grade. Full implementation of the program requires a specific schedule design which allows the student to pursue the enrichment at a rate and sequence appropriate to his/her own intellectual and interest development. Flexibility on the part of teachers charged with this program is essential.

LOGIC STRAND

Embodiments: Logic Blocks, attribute pieces, colored cubes, STAR - a nonverbal language activity, logic board, visual logic game, creature cards, people pieces, analogy games (word and visual), Wif-n-Prof, figure pattern sequences, deductive logic puzzles of identity

Objectives: Students will be able to....

a. use the principles of classification to compare and contrast items and/or events;
b. use statements of negation, of existence, universality, and the connectives of and, or, if....then to describe collections of items or events;
c. use the ordered forms of logical communication to test an argument's validity, complete analogies, generate conclusions, and solve deductive reasoning problems;
d. use the principles and symbols of sentential calculus to construct and describe relations among events or collections of items;
e. use propositional logic to generate or validate conclusions;
f. articulate some of the impacts of logical development in mathematics and technological application.

Topics:

I. Classification
   A. Extensions of descriptions
   B. Intensions of an event or item
   C. Negation
      1. extensions
      2. intensions
   D. Universality, choice, and their negation

II. Set concepts and notations
   A. Member, subset, null set, Venn diagrams
   B. Relations between sets
      1. intersection (and)
      2. union (exclusive vs. inclusive or)
      3. inclusion (if... then)
      4. disjoint (not)
   C. Set descriptions and constructions using similarities/differences or antonyms/synonyms

III. Compound statements, two-valued truth, and logical connectives in language
   A. And
   B. Or
   C. Not and
   D. Not or
   E. Conditicnals

1. implication vs. inference
2. converse
3. contrapositive
4. inverse

F. Analogy
1. pictorial
2. symbolic
a. verbal
b. numerical
c. spatial
3. construction

G. Truth Tables and statements
1. tautalogies
2. contradictions

H. Quantifiers
1. universality
2. existence
3. negation of both

IV. Arguments

A. Validity and rules of inference
B. Generation and validation of arguments
C. Modes of deductive reasoning
D. Paradoxes

V. Impact of Logics
A. Development of Mathematics (logicism,formalism, intuitionism)
B. Probabilistic logic
C. Technology and Planning
1. game theory
2. artificial intelligence
3. computer development

ACTIVITY #1: LOGIC STRAND

SECTION 1B: Intensions of an Event or Item

Objective: Given examples of an event or an item which are 'similar'
a. determine whether other events or items are also 'similar'
b. communicate a description which provides the 'similarity' among the events or items

Materials: Creature Cards
Nonsense Words

Procedure: I. What is a "livi"?

is a livi; so is

is not a livi; likewise is not a livi.

a. Which of these are a livi?

b. Draw an example of a "livi".

c. Tell what makes something a "livi". Is the word 'livi' a livi?

II. What is a "tuoraf"?

The following are examples of a "turoraf".

a comet, the Milky Way, the planet Jupiter

The following are not examples of a "tuoraf"

a sailboat, a Milky Way chocolate, a juniper tree

a. Put a * next to the name of each "tuoraf"

The North Star	helium
super nova	asteroid
snakes	astrodome
Nova car	

b. Give an example of a "tuoraf"

c. Tell when something is a "tuoraf"

The purpose of this activity is to have a student be able to not only distinguish the critical criteria for an event or item to be included in a class, but also generate and validate conclusions concerning other events or items which may be in the class and to communicate what these criteria are.

This classification ability can be operational at the concreate, iconic, or symbolic levels, and could be practiced henceforth at any of the levels.

ACTIVITY #2: LOGIC STRAND

Strand IIIFa: Analogies, verbal

Objective: Given examples of verbal analogies
a. complete verbal analogies
b. generate verbal analogies
c. explicate complete or generated verbal analogies

Materials: The following is an analogy written in simple form. Other analogies may be written in a more complicated manner.

carpenter is to saw as plumber is to wrench.

Shorthand: carpenter: saw :: plumber: wrench

A carpenter uses a tool called a saw. A plumber uses a tool called a wrench.

This is a good relationship sentence. The relationship between a carpenter and the saw is similar to that of a plumber and a wrench.

Procedure: * a. Complete the following as analogies, and explain each with a relationship sentence.

1. book is to paper as shirt is to _____.

2. dam is to flood as _____ is to disease.

3. _____ is to garden as a bumper is to car.

4. mouth is to talk as hand is to _____.

5. 6 is to 2 as 21 is to _____.

b. Each problem is given in a form of a chart. The words which go in the empty columns are listed underneath the chart. Each line of the chart is an analogy by itself. Complete the chart.

*good sources:

Whimbey, Arthur and Jack Lochhead, Problem Solving and Comprehension
Franklin Institute Press, 1979

* 1.

1	2	3	4
down	up		
laugh	cry		
old	new		
sooner	later		

COLUMN 3: basement, before, mouth, stale

COLUMN 4: after, eyes, fresh, roof

2.

1	2	3	4
boy			
man			
man			
niece			
son			

COLUMN 2: boy, girl, man, uncle, woman

COLUMN 3: daughter, girl, husband, nephew, woman

COLUMN 4: aunt, boy, girl, wife, woman

*good sources: Midwest Publication Co., Inc., Pacific Grove, CA
"Verbal Analogies" Books A,B,C. 1980

ACTIVITY #3: LOGIC STRAND

Section IV: Validating an argument

Objective: Given the premises of an argument and different conclusions, determine the validity of each conclusion.

Materials:

1. If the door was closed but unlocked, then the burglar didn't break in. If the door was open but the lock was on, then the burglar didn't break in.

   Conclusions:

   a. If the door was closed and locked, then the burglar broke in.

   b. If the door was open and unlocked, then the burglar broke in.

   c. If the burglar broke in, then either the door was not closed or it was not locked.

Procedure: Use some form of truth analysis to determine validity. Remember validity requires the conjunction of all the premises to imply the conclusion.

COMBINATORICS, PROBABILITY, INTRODUCTION TO STATISTICS STRAND

Embodiments: Logic Blocks, dice, card decks, colored cubes, spinners, slot machines, random number tables, surveys, frequency charts, I Ching, Biorhythm, networks, dominoes, checkers, symbols of an ordered set (alpha-numeric), sources of vital statistics

Objectives: Students will be able to.....

a. use the principles and methods of calulation for permutations and combinations to find discrete probabilities
b. develop tree diagrams or other appropriate charts to describe the probabilities of series of events
c. use the principles of mutually exclusive events, independent events, and conditional events to establish a measure of the probability among events
d. conduct surveys in order to make probabilistic or statistical predictions about specified events
e. use the various measures of central tendency in application
f. use the principles of dispersion in applications

Topics:

I. Methods of counting

A. Set operations and counting
B. Fundamental Counting Principle
1. permutations and the factorial symbol
a. using all with no repeats
b. using all with repeats
c. using some with no repeats
d. using some with repeats
e. in a circle
C. Combinations and the Pascal triangle

II. Probability with equally likely events

A. Experiments
B. Sample and outcome spaces
C. Finite spaces
D. Tree diagrams
E. Types of events
1. mutually exclusive
2. independent
3. replacement vs. non replacement
4. conditional

III. Progressions, sequences, series

A. Arithmetic
B. Geometrics
C. Fibonacci and Lucas

IV. Random drawings, surveys, large sample spaces

ACTIVITY # 1 COMBINATORICS, PROBABILITIES, INTRODUCTION TO STATISTICS

Strand IBIC: Counting with permutations of some unrepeated events.

Objective: To construct the permutations of unrepeated events and to use the factorial principle to calculate the number of permutations.

Materials: Six different colored cubes:

Black, White, Red, Blue, Yellow, Green
B W R A Y G

Complete the following table by actually making all the different ways of placing the number of different colored cubes in a row or column.

Total Number of Blocks	Number of Blocks Used	Number of Different Arrangements
1	1	1
2	2	2
	1	2
3	3	6
	2	
	1	
4	4	24
	3	
	2	
	1	

Total Number of Blocks	Number of Blocks Used	Number of Different Arrangements
5	5	120
	4	
	3	
	2	
	1	

Can you tell how to complete the chart for six different colored blocks?

ACTIVITY # 2 COMBINATORICS, PROBABILITY, INTRODUCTION TO STATISTICS

Strand IIIB: Geometric Sequences

Objectives: To develop different payoff schedules for a pyramid organization

B = any beginning whole number between 1 and 10

R = any multiplier whole number between 1 and 10

Start with any B number you wish; multiply it by any R number you choose, then continue multiplying each product by the R number until you reach the Rth term.

E.g.: B = 3, R = 4

3, 12, 48, 192. Stop; this is the 4th term.

Suppose that each term is the multiplied by a payment.

Cost, C, where $C = |B - R|$ and represents dollars,

Which choice of B and R will generate the greatest payment at the Rth term?

For example in the B = 3, R = 4 case $192 is paid at the fourth term.

ACTIVITY #3: COMBINATORICS/PROBABILITY

STRAND: Expectation and Variability (V and VI)

Objective: To calculate mathematical descriptions of an event space and compare these with the actual event.

Materials: 1 Tetrahedron Die, faces numbered 1, 2, 3, 4

Let the numbers 1, 2, and 3 be considered prime numbers. The die is tossed three times. Let P be the random variable showing the longest string of prime numbered faces to appear. Find the following through expermentation (33 trials) and through calculation:

a. the distribution of P

b. the expectation of P

c. the variance of P

d. the standard deviation of P

ACTIVITY #4: COMBINATORICS/PROBABILITY

STRAND: Types of Events - Conditional (IIE4)

Objective: To develop a method to calculate conditional probabilities

Material: 6 different colored cubes (white, black, green, red, blue, yellow)

Describe all the ways to pick a group of 3 different colored cubes from the collection of 6 cubes.

Arrange the selections in some filing system according to the first cube's color.

Find the following probabilities by listing all the successful events: events:

1. The black cube is in the group of 3.
2. The black cube is and the yellow cube is not in the group of 3.
3. Neither the black nor the yellow cube is in the group of 3.
4. The black, white or yellow cubes are not all in the group of 3.
5. If the black and white are in the group, the yellow cube is not.
6. The red or green or both cubes are in the group of 3.
7. If white-black, red-green, and blue-yellow are chromatic opposite pairs... a chromatic opposite pair in the group of 3.
8. If red-blue-yellow are primary colors ... two or more primary colored cubes are in the group of 3.

ACTIVITY #5: COMBINATORICS, PROBABILITY

STRAND: Fibonacci Sequence (IIIC)

Objective: To generate the sum of squares of the Fibonacci sequence numerical, graphical and through the use of a BASIC Program

Material: Graph Paper

1. Examine the pattern started below and complete it the 10th line

   a. $1^2 + 1^2 = 2 = 1 * 2$

   b. $1^2 + 1^2 + 2^2 = 6 = 2 * 3$

   c. $1^2 + 1^2 + 2^2 + 3^2 = 15 = 3 * 5$

   d. $1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 40 = 5 * 8$

2. Each line has a graphic associated with it. Draw the next 6 figures associated with the continued pattern above.

   a. $1^2 + 1^2 \rightarrow$

   b. $1^2 + 1^2 + 2^2 \rightarrow$

   c. $1^2 + 1^2 + 2^2 + 3^2 \rightarrow$

   d. $1^2 + 1^2 + 2^2 + 3^2 + 5^2 \longrightarrow$?

3. Write a BASIC program to find the sum of squares of the Fibonacci Sequence to the 11th term.

Mentally Gifted Program

GEOMETRY, TOPOLOGY STRAND

Embodiments: Synernetics, Platonic and Archimedian solids, mirrors sphere, geoboards, special topological surfaces, maps natural occurring crystals, physical models of chemical bondings

Objectives: Students will be able...

a. to extend the counting, measuring, and calculating principles of Euclidean recto-linear geometry
b. to use the principles of proof and constructing a metric space to examine other types of geometric spaces
c. to use the principles of invariants under transformations to examine topological properties, affine geometries, and maps
d. to use the principles of matrices in application for specific transformations in a metric space
e. to use the principles and concepts of trigonometry to solve measuring problems and vector applications
f. to consider the concepts of constant width curves, fractals, envelopes, and involutes in application

Topics:

I. Transformations

A. Symmmetry
1. two dimensional figures in the plane or on the sphere
2. polyhedra (linear/ spherical)
3. Miller indices and natural crystals
4. Special solids
5. Constructions in two and three space (linear/spherical)
6. Tiling patterns, tesselations, and space-filling

B. Reflection
1. parallel mirrors
2. intersecting mirrors

C. Rotation
D. Dilations: similarity vs congruence
E. Projections

II. Topology

A. Nodes, networks in surfaces and polyhedra
B. Invariants
C. Exterior vs Interior
D. Special surfaces and spaces and their properties

III. Trigonometry, the right triangle and circularity

A. Sine vs. cosine
B. Application
C. Matrices and transformations in Cartesian Coordinates
D. Matrices and vectors
E. Spherical geometry
F. Envelopes and involutes

IV. Non-Euclidean Geometry

A. Historical development and implications
B. Affine Geometry
C. Map Making
D. Fractals and dimension and chance
E. Celestial Navigation

V. Curves of Constant Width, spirals and black holes

ACTIVITY #1: GEOMETRY, TOPOLOGY

STRAND: Invariants (IIB)

Objective: To become actively familar with topological invariants

Material: Plastaline

Rules of Play:
1. You can not make something hollow by pressing a surface of plastaline to itself
2. You cannot change the number of 'holes' a piece of plastaline

Examples:
1. Start: Rectangular sheet of plastaline, no holes
What can you make?

a. a fork
b. a solid ball
c. empty box with an open flap
d. coffee cup with handle
e. an inner tube
f. a clasped braclet
g. a cup with no handle
h. hollow ball

2. Start: Rectangular sheet of plastaline; 1 hole
What can you make?
Which of the objects in Problem 1a through 1h?

k. a funnel
l. a braclet
m. a hat
n. a spring
o. open bottle
p. a coat hanger
r. a piece of pipe
t. solid doughnut

3. What type of sheet will make these items?

w. a spring
x. pair of scissors
y. a trophy with two handles
z. a tennis rachet with no strings

ACTIVITY #2: GEOMETRY, TOPOLOGY

STRAND: Envelopes and Involutes (IIIF)

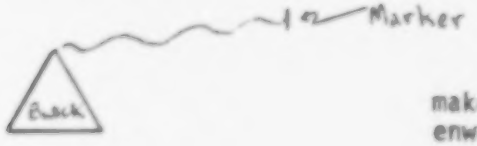
Objective: To produce involutes or envelope by construction

Materials: Drawing Compass
Ruler
String
Logic Blocks

I. Involutes:

Affix any of the six Logic Block shapes to a drawing surface of graph paper and attach one end of a string to its edge. Place a scribe at the other end of the string. Wind the string onto the block and trace the path of the taut string being enwrapped.

e.g.



make string taut and enwrap the block

Repeat for all six shapes. What conclusions can you infer?

II. Envelopes

a. elliptical: Draw rays from any point in the circle. Draw a perpendicular to each ray where it meets the circle.

b. hyperboloid: Draw rays from any point out of the circle. Draw perpendiculars at where the rays intersect the circle.

c. cardeoid: Cut a circle into any even number of parts greater than 6. Number the points starting with 1 to N. Connect every point to its double.

FUNCTION STRAND

Embodiments: Concrete materials for set equivalence, calculators, charts and tables, number-letter codes, number sequences, function machine diagrams, calculators, computers.

Objectives: Students will be able to:

a. Determine one-to-one correspondence between finite sets; equivalence of infinities with infinite sets.
b. Apply functional notation and structure to operations with reals.
c. Investigate functional behavior of operations.
d. Investigate composition of functions
e. Perform arithmetic operations with functions
f. Evaluate single-valued and multiple valued-functions.
g. Operate with single-variable and multiple-variable functions.

Topics:

I. One-to-one correspondence
   A. Finite sets
   B. Equivalence

II. Operations with numbers
   A. Wholes -
      1. Basic operations and their inverses.
   B. Rationals
   C. Reals

III. Closure

IV. Relations
   A. Definition and notation
   B. Non-mathematical relations
   C. Mathematical relations
   D. Properties of relations

V. Roots and Exponents
   A. Inverse operation; functional structure

VI. Function definition, notation
   A. Range-domain
   B. Specific examples

VII. Composition of functions
   A. Operation of "following"
   B. Inverse functions

VIII. Single/multiple valued functions
    A. Evaluation
    B. Determination of inverse
        1. Existent,non-existent

IX. Single/multiple variable functions
    A. Linear relationships
    B. Quadratic relationships
    C. Direct/Inverse variation

X. Algebra of Functions

XI. Other functions
    A. Circular
    B. Trigonometric
    C. Polynomial
    D. Transcendental

ACTIVITY # 1 - FUNCTION STRAND

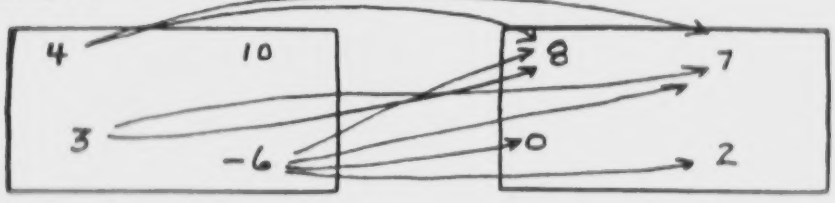
Section IV - Relations

Objective: To have students relate members of two sets according to definition of relation given.

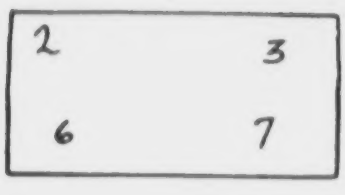
Materials: Paper, pencil

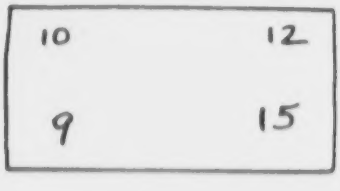
Procedure: Distribute worksheets.Have students connect members of sets according to relation given by drawing arrows.

Sample: Relation - " is less than"

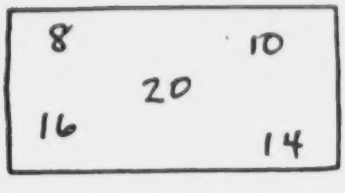


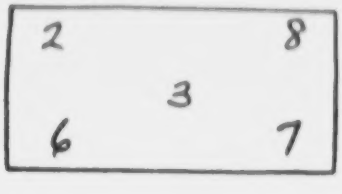
Example 1. Relation - " is a factor of"



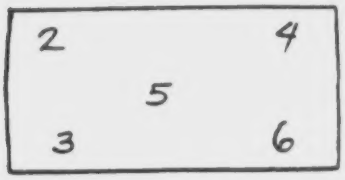


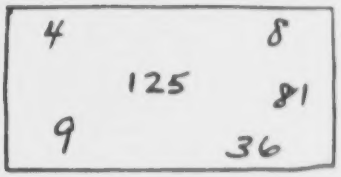
Example 2. Relation - " is a multiple of"



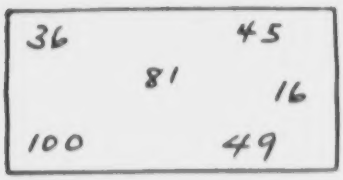


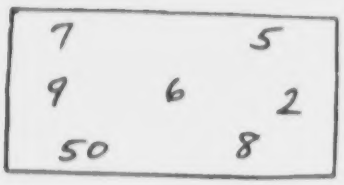
Example 3. Relation - " is a root of"



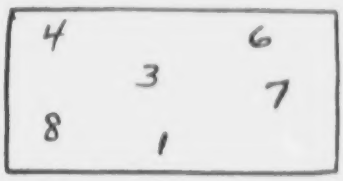


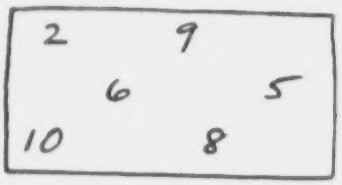
Example 4. Relation - " is the square of"



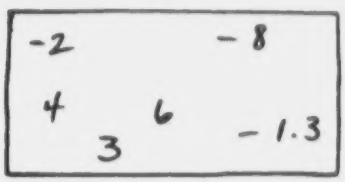


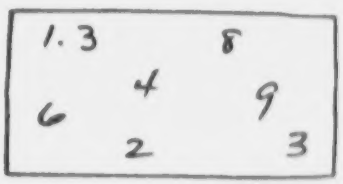
Example 5. Relation " is relatively prime to"





Example 6. Relation - " has the same absolute value as"





ACTIVITY # 2 - FUNCTION STRAND

Section VII-Composition of Functions

Objective: To have students determine the net result of composing two functions by stating the resultant function and determining several values.

Materials: Paper,pencil, calculators

Procedure: Have students complete worksheet using calculators to compute values where necessary.

Sample: $f(x) = 2x + 3$ $g(x) = x^2$. Determine $f(g(x))$ for $x = 2,-3,6$

State $f(g(x))$ as a single function.

$g(2) = 4$, $f(4) = 11$, so $f(g(x)) = 11$
$g(-3) = 9$, $f(9) = 21$, so $f(g(x)) = 21$
$g(6) = 36$, $f(36) = 75$, $f(g(x)) = 75$

$f(g(x)) = 2x^2 + 3$

Example 1. $f(x) = 3x^2$, $g(x) = 2(x-1)$. Compute $f(g(x))$ for $x = 3,-4,6$

Write $f(g(x))$ as a single function.

Example 2. $f(x) = (x+3)^2$, $g(x) = -x$. Determine $g(f(x))$ for $x = -7, 12, 10$

Write $g(f(x))$ as a single function.

Example 3. $f(x) = |x|$, $g(x) = -x^3$. Determine $f(g(x))$ for $x = -2, 5, -6$

Write $f(g(x))$ as a single function

Example 4. $f(x) = \sqrt{x}$, $g(x) = x + 12$. Determine $f(g(x))$ for $x = 37, 88$

Write $f(g(x))$ as a single function.

Example 5. $f(x) = 2x^2 - 3x$, $g(x) = 4 - (x-2)$. Determine $f(g(x))$ for $x = 3,-4,9$

Write $f(g(x))$ as a single function.

PART II

For each of the given functions f(x) and g(x), state both f(g(x) and g(f(x) as single functions:

f(x)	g(x)	f(g(x)	g(f(x)
$3x - 6$	x^2		
$-x + 4$	$x - 5$		
$-x^2 - 2$	$3x$		
$\|x\| - 3$	$x+ 1$		
$2x^2 + x$	$-3x - 1$		
$\frac{1}{x}$	x^2		

ACTIVITY # 3 - FUNCTION STRAND

Section VIII- Inverse Function

Objective: To have students determine the inverse of a given function if it exists, with domain given.

Materials: Charts showing values for given input in a function, with space to write inverse form if it exists and output values.

Procedure: Have students complete charts as shown in sample.

Sample:

x	f(x)	values	$f^{-1}(x)$	values
	$2x-1$		$\frac{x+1}{2}$	
2	→	3	→	2
5	→	9	→	5
-2	→	-5	→	-2

Example 1. domain - reals

x	f(x)	values	$f^{-1}(x)$	values
	$-x + 2$			
5				
-3				
4				

Example 2. domain - reals

x	f(x)	values	$f^{-1}(x)$	values
	$x^2 - 1$			
-8				
10				
1.5				

PART II.

For each of the given functions, determine the inverse if it exists and write it in the box shown.

ALL FUNCTIONS HAVE REAL NUMBERS AS DOMAIN

1. $f(x) = 3x - 5$ $\quad f^{-1}(x) =$ ☐

2. $f(x) = |x|$ $\quad f^{-1}(x) =$ ☐

3. $f(x) = -x^2 + 6$ $\quad f^{-1}(x) =$ ☐

4. $f(x) = \frac{1}{x}$ $\quad f^{-1}(x) =$ ☐

5. $f(x) = x^2 + 2x - 5$ $\quad f^{-1}(x) =$ ☐

6. $f(x) = x(2 - x)$ $\quad f^{-1} =$ ☐

Mentally Gifted Program

COMPUTER STRAND

Embodiments: Simulated computers, flowcharts,micro-computers

Objectives: Students will be able to:

a. List important developments and dates in the history of computers.
b. Describe the basic components of a computer and their function.
c. Define standard acronyms relating to computers such as ROM,RAM, ASCII,etc.
d. Plan for the operation of a computer with regard to a desired outcome through flowcharting.
e. Use BASIC statements in writing simple programs.
f. Format the output of programs using BASIC terminology.
g. Write programs which include logical tests and loops.
h. Write programs which make use of a data base.
i. Load, run, and save programs on tape or disc.
j. Write programs which transfer operation to subroutines.
k. Write programs which use other mathematical operators such as INT,ABS, DEF FNX, etc.
l. Write programs which use arrays, string variables and matrices.
m. Write programs which create simple graphics effects.

Topics:

I. History of computing devices
  A. Ancient devices
    1. Stones
    2. Rods & Sticks
    3. Abacus
    4. Others
  B. Calculating machines
  C. Early computers
  D. Present technology

II. Construction
  A. Input/Output devices
  B. Central processing unit
  C. Memory and storage

III. Flowcharting
  A. Symbols
  B. Construction
  C. Analysis
  D. Debugging

IV. Elementary programming
  A. Variable designation
  B. PRINT

C. INPUT
D. LET
E. Immediate execution mode of computer w/PRINT
F. READ,DATA,RESTORE

V. Formatting
A. Comma;Semicolon
B. PRINT formats;TAB

VI. Logical transfers
A. GOTO
B. IF...THEN
C. GOSUB

VII. Loops
A. Loops with LET
B. Variable Loops
C. Analysis of loops
D. FOR-NEXT
E. Decision loops
F. Nested loops

VIII. Subscripted variables
A. DIM
B. Polynomials
C. Sort algorithms

IX. Matrices
A. MAT
B. Operations with matrices

X. Other operators
A. INT
B. ABS
C. RND
D. DEF FNX
E. SQR

XI. Graphics
A. Low-resolution
B. High resolution
C. HOME-TEXT

SAMPLE ACTIVITY

MENTALLY GIFTED CURRICULUM

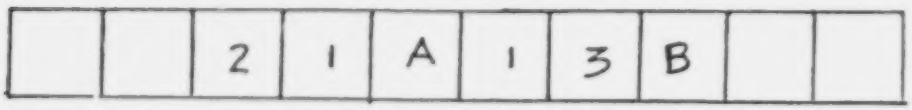
Strand - Computers
Section - III
Title - Paper Computer

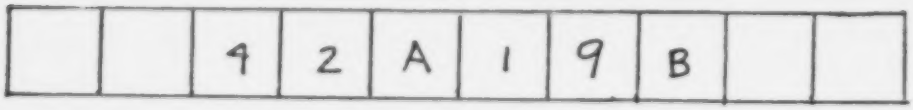
Objectives: Students will follow steps in a flowchart and analyze the process followed.The flowchart will illustrate an incremental method of computing.

Materials: Preprinted worksheets as shown,pencils,erasers.

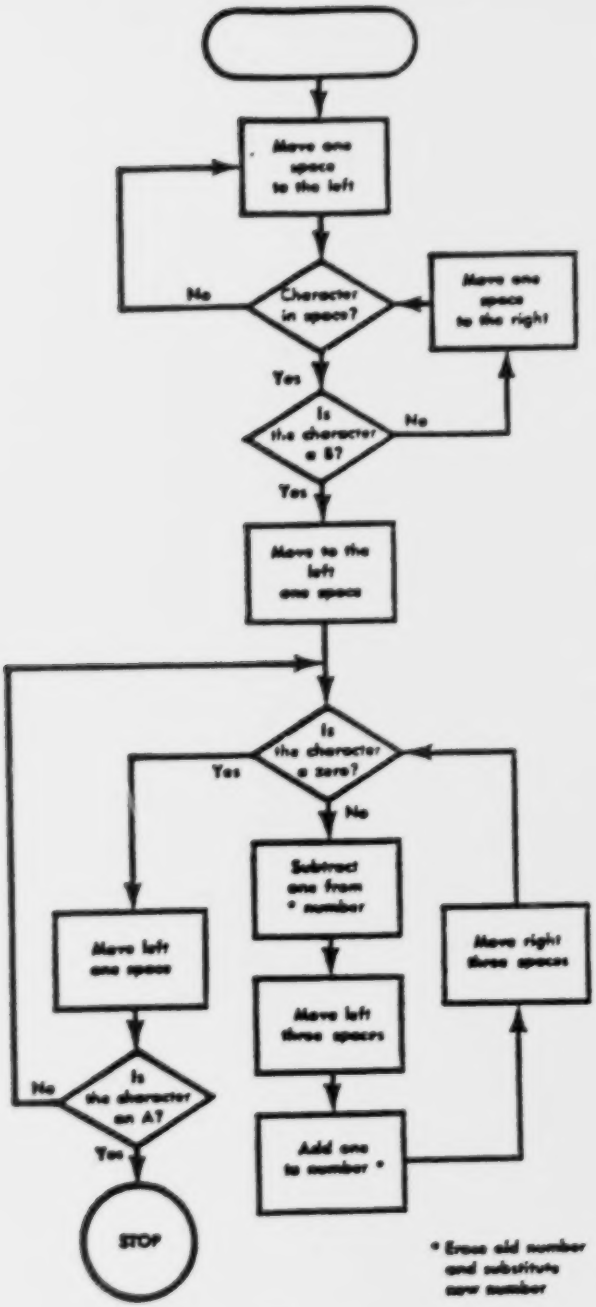
Procedure: Distribute materials to students. Have ach student choose numbers to be written in the proper spaces in the strip as shown. Each student then follows the directions in the flowchart until finished.
The teacher should then discuss the process followed and the results in terms of how a computer works.

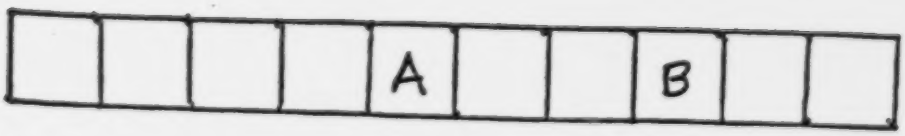
SAMPLE COMPUTATION STRIPS





NOTE THAT "A" AND "B" ARE ALWAYS IN SAME BOXES. THE NUMBERS CHOSEN BY THE STUDENTS (2-DIGIT NUMBERS) ARE ALSO WRITTEN AS SHOWN EACH TIME.


Move one space to the left
Character in space?
No
Move one space to the right
Yes
Is the character a B?
No
Yes
Move to the left one space
Is the character a zero?
Yes
No
Subtract one from * number
Move left one space
Move left three spaces
Move right three spaces
Is the character an A?
No
Yes
Add one to number *
STOP
* Erase old number and substitute new number


A
B

SAMPLE ACTIVITY

MENTALLY GIFTED CURRICULUM

Strand - Computers
Section V
Title - Formatting

Objectives: Students will amend a given program to "format" the output as stated.

Materials: Printed sheets with sample program(s) and directions on what format is needed for output, pencils, computer to check results. *

Procedure: Distribute sheets to students. Have students write necessary lines to amend program as requested. Results should then be checked on computer.

SAMPLE PROGRAM # 1

```
10 INPUT A$
20 INPUT B$
30 INPUT C$
40 PRINT
50 PRINT
60 PRINT A$,B$,C$
```

SAMPLE PROGRAM # 2

```
10 INPUT X,Y
20 INPUT A$,B$
30 PRINT
40 PRINT
50 PRINT "THE SCORE FOR"A$"IS"X
60 PRINT "THE SCORE FOR"B$"IS"Y
```

SAMPLE PROGRAM # 3

```
10 INPUT X,Y,Z
20 LET S = X + Y + Z
30 PRINT
40 PRINT
50 PRINT"FIRST NUMBER";"SECOND NUMBER";"THIRD NUMBER"
60 PRINT X;Y;Z
```

CHANGES

1. Change program #1 so A$,B$ and C$ are printed in three differnt zones.
2. Change program #2 so the first statement is printed in zone 1 and the second statement is printed in zone 2.
3. Change program #3 so "FIRST NUMBER" is printed at column 1,"SECOND NUMBER" is printed at column 15 and "THIRD NUMBER" is printed at column 29. The number values should appear beginning at the same places respectively.

*Activity written for Apple II computer

SAMPLE ACTIVITY

MENTALLY GIFTED CURRICULUM

Strand - Computers
Section X
Title - Monte Carlo Pi

Objective: Students will calculate an approximation of pi using the random number generator and a "Monte Carlo" method of calculating.

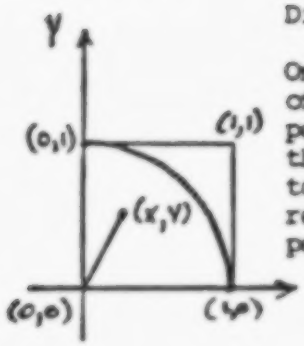
Materials: Computer (activity written for Apple II)

Procedure: Develop with the students the program as shown. This should be illustrated on chalkboard before entering into system. Modifications may be made to the program according to individual needs.

Allow the program to be entered by the students and run. Line 10 should be changed after each run to a higher number of iterations. The second run could be 200, the third, 500, the next 1000 and so on up to 10000 if time permits.

Discuss with the students the apparent changes in the value of PI that are generated by each run.

Discuss also why the method works according to the following:



One quadrant of a unit circle lies within the square. Since the area of this circle is PI, the area of the quadrant show is $\pi/4$. Choosing points (X,Y) at random and computing the distance from the point to the center of the circle will result in lengths less than 1, equal to 1 and greater than 1. Those lengths less than or equal to 1 will represent points inside the circle. The ratio of these points to all points within the square is approximately $\pi/4$: 1. Counting these points after many trials will give an approximation of $\pi/4$. Multiplying this by 4 gives an approximation of π.

PROGRAM -

```
10 For I=1 to 100
20 LET X = RND (1)
30 LET Y = RND (1)
40 LET R = SQR (X∧2 + Y∧2)
50 IF R > 1 THEN 70
60 H = H+1
70 NEXT I
80 PRINT" PI IS APPROXIMATELY ="4*H/I
```

See next page for anecdoted program listing

```
10 for I = 1 to 100
```
(This line determines how many points will be chosen at random)

```
20 LET X = RND(1)
```
(This line chooses a value between 0 and 1 for X at random)

```
30 LET Y = RND(1)
```
(This line chooses a value between 0 and 1 for Y at random)

```
40 LET R = SQR (X∧2 + Y∧2)
```
(This line computes the distance from the randomly chosen point (X,Y) to the origin (0,0) using the Pythagorean Formula)

```
50 IF R > 1 THEN 70
```
(This line transfers control to line 70 each time a length greater than 1 is encountered. This is done because we are only interested in counting those lengths less than or equal to 1, since these represent points inside the circle.)

```
60 H = H + 1
```
(This line is a counter which increases by 1 each time a length less than or equal to 1 is found.)

```
70 NEXT I
```
(This line transfers control back to the beginning of the loop.)

```
80 PRINT" PI IS APPROXIMATELY="4*H/I
```
(This line computes the value of H/I which is the ratio of points inside the circle to the total number of points chosen. This ratio should be close to $\pi/4$ since the area of the quadrant shown is $\pi/4$ and the area of the square shown is 1. When this ratio is then multiplied by 4, the value of π is approximated and printed out here.)

Activity adapted from:
Golden, Neal. Computer Programming in the BASIC Language. Harcourt Brace Jovanovich, New York, 1975.

ABOUT THE AUTHORS

Dr. Alexander Tobin, presently Director of Mathematics Education, K-12 for the School District of Philadelphia, has worked extensively with the gifted and was responsible for initiating the Unified Mathematics Program in Philadelphia, the SMPY Program with Johns-Hopkins University and was a participant in the International Conference on the Teaching of Mentally Gifted in Jerusalem, Israel.

Dr. Tobin has held many offices of distinction, such as, president of ATMOPAV, president of the Pennsylvania Council of Teachers of Mathematics, and president of the National Council of Supervisors of Mathematics. He has been an instructor at Temple University, Beaver College and Antioch.

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* *

Dr. Alan Barson, presently the Director of Chapter I Secondary Mathematics for the School District of Philadelphia, has participated in many institutes and colleges teaching courses for teachers of the gifted. He has been an instructor with Webster College, Syracuse University, Beaver College, Temple University and Marywood College. Dr. Barson also attended the International Conference for the Gifted in Jerusalem, Israel.

Dr. Barson was founder and president of the Pennsylvania Council of Supervisors of Mathematics and Northeast Chairman of the National Council of Supervisors of Mathematics, president of B'nai B'rith Educators Lodge and is presently president of the Pennsylvania Council of Teachers of Mathematics. He has received distinguished service awards from ATMOPAV, PCTM and the Educator of Excellence Award from B'nai B'rith Educators Lodge.

* *

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He has provided in-service education for the Mentally Gifted Program of the Philadelphia Schools for many years, working in the areas of problem solving and computers. He has been a speaker at conferences of the Pennsylvania Association for Gifted Education and has also delivered papers related to gifted education at other local, state and national mathematics conferences. He is on the adjunct faculty of Temple University, Beaver College and Thomas Jefferson University.

* *

Robert D. Murphey is a mathematics education supervisor, grades 1 through 12, within the School District of Philadelphia. He has been investigating the interrelationships among the use and comprehension of native and formal languages, and their effect on constructive acts within mathematics, since school mathematics is concerned with counting, measuring and calculating probabilities or approximations within specified space/time concepts.

As a mathematics graduate of Oberlin College, Dr. Murphey has completed further study and is professionally active in the areas of mathematics education, supervision, and curriculum theory/development. He has taught at grades 6 through undergraduate mathematics in urban settings, alternative schools, and in an international school. His contribution to this publication centered on the programmatic concerns, and leaves the translations of its content to the professionals whose responsibility it is who will provide responsive feedback regarding the program in light of their audiences.